Durham Symposium 2023: Spectral gaps

Welcome to Durham!

All talks will be held on the ground floor of the Mathematics and Computer Science Building in MCS0001.

Lunch is provided by the symposium in the dining room of Collingwood college. There is a drinks reception Monday evening in Collingwood college bar, and there is a banquet dinner on Wednesday evening at Hatfield College dining room. Breakfast and dinner are provided for guests in Collingwood College dining hall (with the exception of Wednesday evening). See the schedule for timings and locations of these and additional refreshment breaks.

There is extra space for working in the common areas on the second and third floors of the building, please feel free to use these. The building can be accessed 9am—6pm.

We will have an optional problem session on Wednesday afternoon — if you'd like to present a problem let one of the organizers know (or just turn up ready).

Wifi can be accessed in the MCS building via eduroam or the network TheCloud@Durham (requires creating an account).

Reimbursement: if you have been offered travel funding, please submit the reimbursement form available at https://www.mmagee.net/DS_SG.html directly to Suzanne Joyce at maths.finance@durham.ac.uk along with electronic copies of your receipts.

MCS0014 = Flexible event space on bottom floor of math department

MCS0001 = Lecture room on bottom floor of math department

CW = Collingwood

Title and Abstracts

Monday 7th August

08:00-9:45	*Breakfast*	CW dining room
9:30-10:00	*Refreshments*	MCS0014
10:00-10:05	Welcome	MCS0001
10:05-11:05	Will Hide	MCS0001
11:15-12:15	Wenyu Pan	MCS0001
12:30-13:30	*Lunch*	CW dining room
14:15-15:15	Alex Gamburd	MCS0001
15:15-16:00	*Refreshments*	MCS0014
16:00-17:00	Nikhil Srivastava	MCS0001
17:30-19:00	*Drinks Reception*	CW bar
19.00-20.00	*Dinner*	CW dining room

Tuesday 8th August

08:00-9:45	*Breakfast*	CW dining room
9:30-10:00	*Refreshments*	MCS0014
10:00-11:00	Petr Kravchuk	MCS0001
11:15-12:15	Dalimil Mazac	MCS0001
12:30-13:30	*Lunch*	CW dining room
14:15-15:15	Ursula Hamenstadt	MCS0001
15:15-16:00	*Refreshments*	MCS0014
16:00-17:00	Ori Parzanchevski	MCS0001
19.00-20.00	*Dinner*	CW dining room

Wednesday 9th August

08:00-9:45	*Breakfast*	CW dining room
9:30-10:00	*Refreshments*	MCS0014
10:00-11:00	Urban Jezernik	MCS0001
11:15-12:15	Sean Eberhard	MCS0001
12:30-13:30	*Lunch*	CW dining room
14:15-15:15	Peter Sarnak	MCS0001 (broadcast)
15:15-16:00	*Refreshments*	MCS0014
16:00-17:00	Problem/discussion session	MCS0001
18:00-23:00	*Hatfield College bar open*	Hatfield College bar
19:30-22:00	*Banquet Dinner*	Hatfield College Dining Room

Thursday 10th August

08:00-9:45	*Breakfast*	CW dining room
9:30-10:00	*Refreshments*	MCS0014
10:00-11:00	Sam Edwards	MCS0001
11:15-12:15	Nina Zubrilina	MCS0001
12:30-13:30	*Lunch*	CW dining room
14:15-15:15	Bram Petri	MCS0001
15:15-16:00	*Refreshments*	MCS0014
16:00-17:00	Anshul Adve	MCS0001
19.00-20.00	*Dinner*	CW dining room

Friday 11th August

08:00-9:45	*Breakfast*	CW dining room
9:30-10:00	*Refreshments*	MCS0014
10:00-11:00	Julia Slipantschuk	MCS0001
11:15-12:15	Mikhail Belolipetsky	MCS0001
12:30-13:30	*Lunch*	CW dining room
14:15-15:15	Yunhui Wu	MCS0001 (broadcast)
15:15-16:00	*Refreshments*	MCS0014
17:30-19:00	Dinner	CW dining room

Titles and abstracts

Anshul Adve (Princeton)

A spectral gap for spinors on hyperbolic surfaces

I'll describe a sequence of hyperbolic surfaces with a choice of spin structure and with genus going to infinity, such that the bottom eigenvalue of the Laplacian on spinors is uniformly bounded below. The surfaces can be taken to be arithmetic. A fun feature of the construction is that it makes use of four different perspectives on Riemann surfaces: algebraic, complex analytic, geometric, and arithmetic. Along the way we'll see that when the Riemann surface in question is sufficiently symmetric, one can pass between these perspectives in an explicit and fruitful manner. This is joint with Vikram Giri.

Mikhail Belolipetsky (IMPA)

Geometric expanders

In a joint work with Hannah Albert we showed that closed arithmetic hyperbolic 3-dimensional orbifolds with larger and larger volumes give rise to triangulations of the underlying spaces whose 1-skeletons are harder and harder to embed nicely in Euclidean space. This is achieved by generalizing an inequality of Gromov and Guth to hyperbolic n-orbifolds and finding nearly optimal geodesic triangulations of arithmetic hyperbolic 3-orbifolds. The relation between the euclidean embeddings and expander graphs goes back to the pioneering work of Kolmogorov and Barzdin from the 1960s. In the talk I will discuss these results with an emphasis on some related open problems.

Sean Eberhard (Queen's University Belfast)

Babai's conjecture for classical groups with random generators, part 2

Consider $G = \mathrm{SL}_n(q)$ acting on $V \setminus \{0\}$, where $V = F_q^n$ is the defining module, and consider the Schreier graph defined by k random elements $x_1, \ldots, x_k \in G$. In this talk we will show that the there is a spectral gap with high probability provided that k is larger than a constant depending only on q. More precisely we will show that the second largest eigenvalue of the normalized adjacency matrix is bounded by $(1 + o(1))q\sqrt{(2k-1)}/k$. The argument is based on the classical trace method approach of Broder and Shamir (1987), suitably adapted to the linear setting. Analogous results hold for other classical groups and for the action on r-tuples of vectors for bounded r, which is a linear analogue of a result for S_n of Friedman, Joux, Roichman, Stern, and Tillich (1998). Time permitting we will also outline these generalizations. As an application we prove Babai's conjecture on the diameter of simple classical groups for generating sets which include both a bounded-degree element and sufficiently many random generators. Joint work with Urban Jezernik.

Sam Edwards (Durham University)

The bottom of the L^2 spectrum of higher-rank locally symmetric spaces

For a rank one geometrically finite locally symmetric space $\Gamma\backslash X$, the bottom of the L^2 spectrum of the Laplace operator is a simple eigenvalue corresponding to a positive eigenfunction if and only if the critical exponent of Γ is strictly greater than half the volume entropy of X. In particular, there exist infinite volume rank one locally symmetric spaces with square integrable positive Laplace eigenfunctions. In contrast, a higher-rank symmetric space $\Gamma\backslash X$ without rank one factors has a square integrable positive Laplace eigenfunction if and only Γ is a lattice. We will explain some aspects of the connection between square integrability of positive Laplace eigenfunctions and Patterson–Sullivan and Bowen–Margulis-Sullivan measures in the higher-rank setting. Based on joint work with Oh and Fraczyk–Lee–Oh.

Alex Gamburd (CUNY Graduate Center)

TBC

Ursula Hamenstädt (University of Bonn)

Spectral gap and eigenfunctions on hyperbolic 3-manifolds

We summarize what is known about the spectral gap and properties of the corresponding eigenfunctions for (possibly not closed) hyperbolic manifolds. We also develop a rather speculative picture on the relation between spectral gap, volume and Heegaard genus as it arises in progress on effective hyperbolization. This is in part joint work with Gabriele Viaggi, and joint work with Elia Fioravanti, Frieder Jaeckel and Yongquan Zhang.

Will Hide (Durham University)

Spectral gaps for random covers of hyperbolic surfaces

Based on joint work with Michael Magee. We study the spectrum of the Laplacian for finite-area hyperbolic surfaces, focusing on the spectral gap, i.e. the smallest non-zero element of the spectrum. The spectral gap can be viewed as a measure of how highly connected a surface is, providing control over its diameter and Cheeger constant. It controls the rate of mixing of the geodesic flow and error terms in geodesic counting. For large genus compact surfaces, $1/4$ is the asymptotically optimal spectral gap.

In a close analogy with results for random regular graphs, we study uniformly random Riemannian coverings of a fixed non-compact finite-area hyperbolic surface X. We show that for any $\epsilon > 0$, a random degree-n cover has no Laplacian eigenvalues below $1/4 - \epsilon$ — other than those of X and with the same multiplicity, with probability tending to 1 as n tends to infinity. As a consequence, we are able to prove the existence of sequences of compact surfaces with genus tending to infinity and spectral gap tending to $1/4$, confirming a conjecture of Buser. I will discuss some ideas of the proofs as well as some further questions.

Urban Jezernik (University of Ljubljana)

Babai's conjecture for classical groups with random generators, part 1

The diameter of a finite group G equipped with a generating set S is the smallest number k so that every element of G can be written as a product of at most k elements from S. Babai's conjecture predicts that diameters of finite simple groups should be polylogarithmic in the size of the group. Much is known about this conjecture, yet it is still open for the family of groups $\mathrm{PSL}_n(F_q)$ as n tends to infinity and q is fixed. We investigate this family in the generic situation when the generating sets consist of random elements and prove that Babai's conjecture holds in this case. Our argument naturally splits into two parts: first we show how to quickly get a transvection using random elements, and second we use spectral gaps to quickly get all other transvections and complete the proof. In this talk, we will gently introduce the problem, outline the strategy of our argument for $\mathrm{PSL}_n(F_q)$, and give some details on the first part of the proof. Joint work with Sean Eberhard.

Petr Kravchuk (King's College London)

In this talk I will review an approach to deriving upper bounds on the gap in the Laplace spectrum of hyperbolic 2-orbifolds, inspired by numerical bootstrap methods from conformal field theory. As I will explain, these bounds are obtained from consistency conditions for integrals of quadruple products of holomorphic differentials and their derivatives, and can be numerically very close to what is believed to be the largest gap. This might suggest that the bounds can be improved to be tight, but I will provide strong evidence that this is impossible. Based on work with Dalimil Mazac, Sridip Pal and Alexander Radcliffe.

Dalimil Mazac (IAS Princeton)

Spectral bounds on hyperbolic 3-manifolds from the associativity of multiplication

I will discuss bounds on the low-energy spectra of Laplacians on compact hyperbolic 3-manifolds and orbifolds. The key ingredient is an infinite collection of identities satisfied by 1) the spectra of the Laplacians on powers of the cotangent bundle, and 2) the integrated triple products of the corresponding eigenfunctions. These spectral identities follow from the consistency of associativity of multiplication of functions on $\Gamma\backslash \mathrm{PSL}_2(\mathbb{C})$ with the spectral decomposition. They can be turned into spectral bounds via linear programming. I will compare the bounds with the spectra of various hyperbolic 3-manifolds and show that they are often nearly sharp. Finally, I will explain how this approach is related to the conformal bootstrap program in conformal field theory, which served as the original inspiration. Based on work with J. Bonifacio, P. Krachuk, and S. Pal.

Wenyu Pan (University of Toronto)

Exponential mixing of frame flows for geometrically finite hyperbolic manifolds

The frame bundle of an n-dimensional hyperbolic manifold X is the homogeneous space $\Gamma\backslash \mathrm{SO}(n,1)^o$ for some discrete subgroup Γ and the frame flow is given by the right translation action by a one-parameter diagonalizable subgroup. We assume that Γ is Zariski dense and X is geometrically finite, i.e., it need not be compact but has at most finitely many ends consisting of cusps and funnels. We endow the frame bundle with the unique probability measure of maximal entropy called the Bowen-Margulis-Sullivan measure. In joint work with Jialun Li and Pratyush Sarkar, we prove that the frame flow is exponentially mixing. The proof uses a countably infinite coding and Dolgopyat's method à la Sarkar–Winter and Tsujii–Zhang. To overcome the difficulty in applying Dolgopyat's method due to the cusps of non-maximal rank, we prove a uniform large deviation property for symbolic recurrence to certain large subsets of the limit set of Γ.

Ori Parzanchevski (Hebrew University of Jerusalem)

Random walks on Ramanujan graphs and complexes

Recent works of Lubetzky-Peres, Sardari and Nestoridi-Sarnak have shown that non-backtracking random walks on Ramanujan graphs exhibit optimal cutoff; namely, the L^1-mixing time of the walk is around $\log_k(N)$ steps (for a k-regular walk on N vertices). I will explain these results, their generalization to Ramanujan complexes of arbitrary dimension, and consequences for optimal expansion in finite simple groups.

Bram Petri (IMJ-PRG and Sorbonne Université)

Linear programming bounds for hyperbolic surfaces

I will speak about extremal problems in the (spectral) geometry of hyperbolic surfaces and how linear programming methods based on the Selberg trace formula can help. I will not assume any familiarity with hyperbolic geometry or the Selberg trace formula. This is joint work with Maxime Fortier Bourque.

Peter Sarnak (Princeton University and IAS Princeton)

Prescribing the spectrum of locally homogeneous geometries

We review some recent developments (conformal bootstrap and random covers) concerning the bass-note spectrum of invariant operators on locally homogeneous geometries and in particular hyperbolic manifolds and of large 3-regular graphs. Of special interest are rigidity features for creating spectral gaps.

Julia Slipantschuk (University of Warwick)

Resonances for Anosov diffeomorphisms on the torus

I will present a complete description of Pollicott-Ruelle resonances for a class of rational Anosov diffeomorphisms on the two-torus. This allows us to show that every homotopy class of two-dimensional Anosov diffeomorphisms contains (non-linear) maps with the sequence of resonances decaying stretched-exponentially, exponentially or having only trivial resonances.

Nikhil Srivastava (UC Berkeley)

Many Nodal Domains in Random Regular Graphs

A nodal domain of a Laplacian eigenvector of a graph is a maximal connected component where it does not change sign. Sparse random regular graphs have been proposed as discrete toy models of “quantum chaos", and it has accordingly been conjectured by Y. Elon and experimentally observed by Dekel, Lee, and Linial that the high energy eigenvectors of such graphs have many nodal domains.

We prove superconstant (in fact, nearly linear in the number of vertices) lower bounds on the number of nodal domains of sparse random regular graphs, for sufficiently large Laplacian eigenvalues. The proof combines two different notions of eigenvector delocalization in random matrix theory as well as tools from graph limits and combinatorics. This is in contrast to what is known for dense Erdos–Renyi graphs, which have been shown to have only two nodal domains with high probability.

Joint work with Shirshendu Ganguly, Theo McKenzie, and Sidhanth Mohanty.

Yunhui Wu (Tsinghua University)

Recent developments on random hyperbolic surfaces of large genus

In this talk, we report several very recent asymptotic results on certain classical geometric quantities viewed as random variables on the moduli space of Riemann surfaces for large genus (and many cusps). This talk is based on several joint works with Hugo Parlier, Xin Nie, Yang Shen and Yuhao Xue.

Nina Zubrilina (Princeton University)

Root Number Correlation Bias of Fourier Coefficients of Modular Forms

Limiting distributions of spectral parameters of modular forms, in the prime, weight, and level aspects, are all well known to have mean 0. Nonetheless, it was recently discovered by Sutherland that when one restricts to forms of a prescribed root number, there are gaps between the means of such averages and 0. These computations extend a recent machine learning based study of He, Lee, Oliver, and Pozdnyakov, who observed a striking oscillating pattern in the average value of the P-th Frobenius trace of elliptic curves of prescribed rank and conductor in an interval range. In my talk, I will point to a source of this phenomenon in the case of all forms and compute the correlation function exactly.